Climate stress tests on corporate bonds

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Effects of climate change are increasingly noticeable, and notably impacts on economic activity. Economic scenarios representing the future possible states of economies are at the core of the regulatory calculations performed by insurance companies. This paper presents and discusses several methodologies that can be employed to integrate the climate risk into the derivation of future scenarios of corporate spreads and probabilities of defaults.

Benchmark method

The first approach we discuss is the method described by the Banque de France in its working paper (see CRSFSA in References). In this paper, this method is called the "benchmark" one. In the rest of the document, the method is adapted to the current framework of the present study.

The event of bankruptcy is described by a random variable D that can take two values: 1 in case of default, 0 otherwise. D is thus described by a Bernoulli distribution of parameter p to be determined. The particularity of the method lies in the fact that this Bernoulli distribution is a *conditional* distribution. Namely, the parameter p defining the distribution expresses as a function of a vector state variable $\tilde{X}$[1]:

$$\mathbb{P}\left(\mathrm{D}=1\middle|\tilde{\mathrm{X}}\right)=1-\mathrm{p}\left(\tilde{\mathrm{X}}\right),\mathbb{P}\left(\mathrm{D}=0\middle|\tilde{\mathrm{X}}\right)=\mathrm{p}\left(\tilde{\mathrm{X}}\right).$$

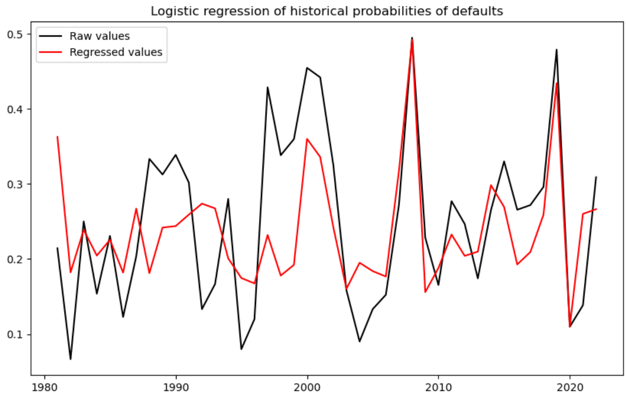
To estimate the function p, a logistic regression is performed:

$$1-\mathrm{p}\left(\tilde{\mathrm{X}}\right)=\frac{1}{1+\exp\left(\beta_0+\widetilde{\beta_1}\cdot\tilde{\mathrm{X}}\right)}.$$

To illustrate this approach, a regression has been made using the historical dataset of probabilities of default coming from the annual study of Standard & Poors Global (S&P Global). See SP2023 in References.

The state process we have chosen is composed of the three following macroeconomic variables that are the evolutions of the French gross domestic product (GDP), the price of oil and the S&P500 index: $\tilde{X}=(\Delta\mathrm{GDP},\Delta\mathrm{Oil},\Delta\mathrm{S\&P})$. The regression is displayed in Figure 1 and is fairly satisfactory: the R^2 is of 36.5%; the global significance of the model is of 0.056%; the respective significance of each coefficient is 0.074, 0.34, 0.011. In particular, the variations of oil prices are not statistically significant in the present experiment.

FIGURE 1: LOGISTIC REGRESSION OF HISTORICAL PROBABILITIES OF DEFAULTS USING THE "BENCHMARK" METHOD



To further develop the integration of climate risk in corporate bonds, we investigate in the remaining part of this paper two processes that have additional abilities compared to the benchmark methodology. Both approaches rely on structural modelling of the company that consider in a single framework the modelling of corporate debt and equity. The first method is a statistical procedure that allows us to model the transition of credit ratings and their associated probabilities in addition to the probabilities of defaults. The second described approach allows us to consistently derive call option prices from corporate bonds, and vice versa. One of the main motivations to consider these approaches is the following: a number of climate stress tests have been proposed by several national regulators (mainly in Europe); they rely on scenarios of the future evolution of some economic quantities. It turns out that the implemented processes of the valuation of insurance undertakings do need a scenario on extra quantities, which are not included in these scenarios. Deriving consistent scenarios on implied volatilities from a given scenario on corporate bonds (as will be illustrated in this paper) is therefore of interest.

1. Symbol ~ refers to multidimensional quantities (vectors).

A real-world approach based on Merton-Vasicek modelling

OVERVIEW OF THE MODEL

Under this approach, we rely on a framework closer to the original Merton model. Specifically, we follow the Merton-Vasicek model, which is a structural model assuming a Gaussian representation of the value of a company, allowing for a deduction of its loan credibility (credit rating, likelihood of default). In its original version (see Merton in References), the Merton model represents the asset value of a company using a lognormal process (as in the celebrated Black-Scholes model used for equity derivatives pricing) and models the event of default as being the moment when the value of the company falls below a threshold. This framework has the main advantage of providing closed-form formulas for several quantities of interest (probabilities of default, distance to default, equity derivatives etc.).

The Merton-Vasicek extension of the model (see Vasicek in References) proposes to represent the asset value of the considered company by a random variable that is based on the sum of an idiosyncratic term and a systemic risk factor. It preserves the advantage of providing a number of analytical formulas, still taking advantage of the Gaussian property of the involved random variables.

We base our framework of analysis on this particular framework. In addition, the model is extended to take into account the fact that the rating of the considered undertaking may vary through time: the asset values of the undertaking do also determine its credit rating, and not only its default. To this end, thresholds are calibrated so that they determine the rating of the company. The smallest calibrated threshold corresponds to the default threshold.

In equation, let us consider a company C whose credit rating is $R(t)$ at time t (typically, $R(t)$ takes values in $\{\mathrm{AAA}, \mathrm{AA}, \mathrm{A}, \mathrm{BBB}, \mathrm{BB}, \mathrm{B}, \mathrm{CCC}\}$) and asset return at time t is denoted by $y^C(t)$. The thresholds determining the current rating of the company are denoted by d_R: if $y^C(t) \in \{d_{R-1}, d_R\}$, then the company is rated $R-1$ at time t (here $R-1$ denoted the credit rating just below R in the increasing order); the extreme events determine when the company takes the best possible rating or when the company defaults. E.g., if $y^C(t) \in \{d_{\mathrm{BBB}}, d_{\mathrm{A}}\}$, then the company is rated BBB; if $y^C(t) \geq d_{\mathrm{AAA}}$, the company is rated AAA and if $y^C(t) \leq d_{CCC}$, the company defaults (or has default) at time t. In particular, the default can be seen as being a particular rating, say $R = D$. Because $y^C(t)$ is assumed to be a Gaussian variable, the probability of the transition of ratings from rating R i.e., assumed to be the initial rating, $R(0) = R$, to rating R' expresses as:

$$
\begin{aligned}
\mathbb{P}(\mathrm{R} \rightarrow \mathrm{R'} \text{ at time t}) = {} & \mathbb{P}(\mathrm{R(t)} = \mathrm{R'}) \\
& = \mathbb{P}\left(\mathrm{d_{R'}} \leq \mathrm{y^C(t)} \leq \mathrm{d_{R'+1}}\right) \\
& = \Phi(\mathrm{d_{R'+1}}) - \Phi(\mathrm{d_{R'}})
\end{aligned}
$$

where Φ is the cumulative distribution function of the normal distribution.

The process y^C is defined as:

$$
\mathrm{y^C(t)} = \sqrt{\rho}\mathrm{X_t} + \sqrt{1-\mathrm{P}}\,\mathrm{Z_T^C}
$$

where X_t is a stochastic process representing the systemic macroeconomic risk, Z_t^C is an idiosyncratic factor representing the specific risk and ρ is a coefficient lying in $]0,1[$ that is the correlation coefficient between asset return and systemic factor.

For simplicity, Z_t^C is assumed to be a Gaussian variable independent of any other source of risk (in particular, Z_t^C and Z_s^C are independent for $t \neq s$). At time t, we can express the conditional transition probabilities as functions of the realisation of the systemic factor and the correlation coefficient:

$$
\begin{aligned}
\mathbb{P}(\mathrm{R} \rightarrow \mathrm{R'} \text{ at time t } |\mathrm{X_t}) = {} & \mathbb{P}(\mathrm{R(t)} = \mathrm{R'}|\mathrm{X_t}) \\
& = \mathbb{P}(\mathrm{d_{R'}} \leq \mathrm{y^C(t)} \leq \mathrm{d_{R'+1}}|\mathrm{X_t}) \\
& = \mathbb{P}\left(\mathrm{d_{R'}} \leq \sqrt{\rho}\mathrm{X_t} + \sqrt{1-\mathrm{P}}\,\mathrm{Z_T^C} \leq \mathrm{d_{R'+1}}|\mathrm{X_t}\right) \\
& = \Phi\left(\frac{\mathrm{d_{R'+1}} - \sqrt{\rho}\mathrm{X_t}}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{\mathrm{d_{R'}} - \sqrt{\rho}\mathrm{X_t}}{\sqrt{1-\rho}}\right) \\
& =: \mathrm{f_{(R,R')}}(\mathrm{X_t}, \rho).
\end{aligned}
$$

The systemic risk factor is meant to represent the overall state of the economy. In the present work, we have performed a statistical analysis to determine the best variable for establishing a model for the evolution of the process X. We propose to write evolution of the systemic risk factor between two dates as

$$
\mathrm{X_{t+1}} = \gamma + (1+\alpha)\mathrm{X_t} + \tilde{\beta} \cdot \widetilde{\mathrm{Y_{t+1}}} + \tilde{\nu} \cdot \tilde{\mathrm{Y}}_{\mathrm{t}} + \epsilon_{\mathrm{t+1}}, (\mathrm{E_1})
$$

where α, γ are constant coefficients, $\tilde{\beta}, \tilde{\nu}$ are multidimensional vectors of coefficients, ϵ_t is a Gaussian variable, independent of all other sources of risk and $\tilde{Y}$ is a vector of variables representing the evolution of the macroeconomic variable; when projecting the state process X, the variable $\tilde{Y}$ should be also be simulated and this can be done with the help of macroeconomic scenarios on future evolution of the economy.

CALIBRATION

First, the thresholds d_R can be estimated using a time series of observed transition of rating: an historical estimation, denoted $\hat{P}_{R\to R'}$, of the probabilities of going from a rating R to a rating R' can be constituted thanks to which the thresholds can be determined iteratively as:

$$d_{R'} = \Phi^{-1}\left(\Phi(d_{R'+1}) - \hat{P}_{R\to R'}\right).$$

Subsequently, the correlation coefficient ρ and the macroeconomic process X need to be determined. The described calibration method is divided into two steps: first, we consider that the values taken by the state process X are parameters, which will be calibrated simultaneously with the other model parameters; in a second step, the parameters that define this state process X will themselves be calibrated, using the values obtained during the first step. Namely, say that on historical period (year, month etc.) t, the process X took the value x_t. It has been observed on this period that a number $n_{R,R'}(t)$ of companies have been going from rating R to rating R'. The *conditional* (to the systemic factor) distribution of the random variables $N_{R,R'}$ counting the number of transitions is modelled by a multinomial distribution:

$$\begin{aligned}&\mathbb{P}\left(N_{R,AAA} = n_{R,AAA,t}, \dots, N_{R,D} = n_{R,D,t} | X_t = x_t\right)\\ &= \frac{n_t!}{n_{R,AAA,t}! \times \dots \times n_{R,D,t}!} F_{(R,AAA)}(X_T, P)^{n_{R,AAA,t}} \times \dots\\ &\times F_{(R,D)}(X_T, P)^{n_{R,D,t}}\end{aligned}$$

where $n_t = n_{R,AAA,t} + \cdots + n_{R,D,t}$ is the total number of observed events.

By independence of the conditional transition distributions, and applying the Bayes formula, we can finally express the likelihood of the observed sample at period t as

$$\begin{aligned}&\mathbb{P}\left(N_{R,AAA} = n_{R,AAA,t}, \dots, N_{R,D} = n_{R,D,t}, X_t = x_t\right)\\ &= \frac{e^{-\frac{x_t^2}{2}}}{\sqrt{2\pi}} \prod_R \frac{n_t!}{n_{R,AAA,t}! \times \dots \times n_{R,D,t}!} F_{(R,AAA)}(X_T, P)^{n_{R,AAA,t}}\\ &\times \dots \times F_{(R,D)}(X_T, P)^{n_{R,D,t}}.\end{aligned}$$

Numerically, to determine the values of ρ and $(x_t)_{t=1,..,T}$, the log-likelihood of the sample is evaluated and minimised to determine the optimal values of correlation coefficient ρ and systemic factor $(x_t)_{t=1,..,T}$:

$$\begin{aligned}L(\rho, x_1, \dots, x_T) = -\frac{T}{2}\log(2\pi) - \sum_{t=1}^{T} \frac{x_t^2}{2}\\ + \sum_{t=1}^{T}\left(\sum_R \log\left(\frac{n_t!}{n_{R,AAA,t}! \times \dots \times n_{R,D,t}!}\right)\right.\\ \left. + \sum_{R'} n_{i,j,t} \log\left(F_{(R,R')}(\rho, x_t)\right)\right). (E_L)\end{aligned}$$

In a second time, we can calibrate the coefficients α, γ, $\tilde{\beta}$ and $\tilde{\nu}$ by performing a linear regression analysis and obtain coefficient estimations through least-squares minimisation.

NUMERICAL EXPERIMENTS

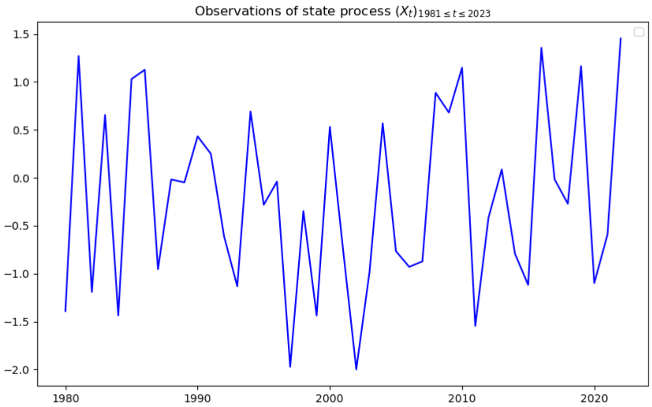
We have applied this statistical approach to two sets of data: a very global set of data representing the global economy and a very specific example of French agricultural data. Due to a lack of relevant historical data, we have applied the considered modelling framework to the case where there are only two ratings classes: default and non-default. In other words, the setting we illustrate focusses on default probabilities, and does not consider transition of rating. However, with proper data, it can be fully extended following the same methodology as described above.

Global data

To calibrate the model, we have used historical series of probabilities of defaults provided by S&P Global in SP2023 (see references). After analysis, we have chosen to link the state process X to to the French GDP (source: INSEE[2]). The historical time series we work with are annual between 1981 and 2022; they gather the bankruptcy of the undertakings in seven different groups of ratings ($AAA, AA, A, BBB, BB, B, CCC$). The minimisation of the log-likelihood provides an estimation of the correlation coefficient $\rho = 0.1$; however, this value is equal to the lower bound set in the minimisation algorithm (for X to be significant in the formulas, we have set a nonzero lower bound during the minimisation routine). Observations of the latent process X are displayed in Figure 2.

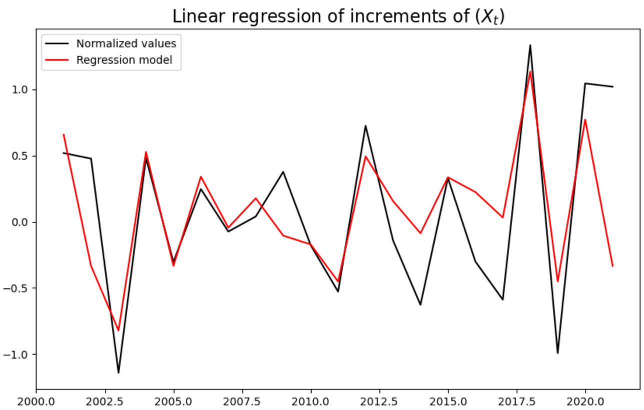
2. INSEE (31 May 2024). The Nation's Accounts in 2023. Retrieved 28 October 2024 from https://www.insee.fr/fr/statistiques/8068582?sommaire=8068749.

FIGURE 2: STATE PROCESS X



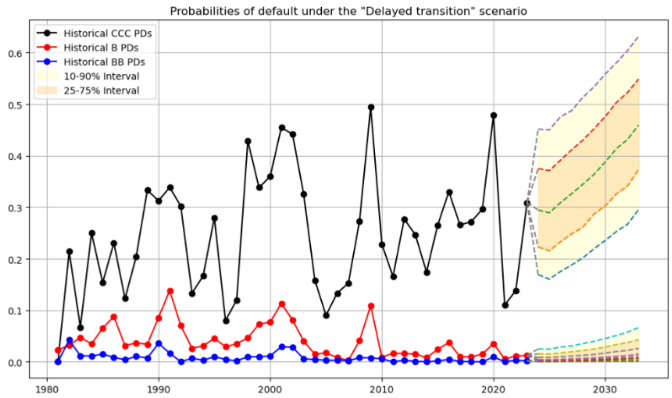
The calibrated values for the state process X justify the choice of the regression model given in Equation (E_1) because a strong non-positive autocorrelation can be observed. In the present experiment, we have set $\gamma = 0$. Performing a least-squares regression on the increments of the state process with the French GDP being the explanatory variable $\tilde{Y}$ provides the following results: $\text{MSE} = 0.86$ and $R^2 = 42.46\%$. The p-values associated with the statistical tests of significance for the coefficients α and $\tilde{\beta}$ (in this case, $\tilde{\beta}$ is a number, not a vector) are, respectively, 0.0% and 5.5%.

FIGURE 3: LINEAR REGRESSION OF THE INCREMENTS OF THE STATE PROCESS WITH RESPECT TO FRENCH GDP



Now that the model is calibrated, we can simulate it and project values of the probabilities of defaults. To do so, we need a scenario for the future evolution of the macroeconomic process $\tilde{Y}$. We have reused the projections of GDP established by the French Prudential Supervision and Resolution Authority, the *Autorité de contrôle prudentiel et de résolution* (ACPR), in its second climate stress test[3] (whose results were published in May 2024[4]). In particular, we have selected the scenario "Delayed transition," in which policies aiming to counter the effects of climate change are not immediately implemented. The starting values of the simulations of process X is taken equal to its last observed values X_{2023}; over the period of projection, it is assumed that the default threshold d_D is constant and equal to its last observed value. We perform this simulation for the three worst ratings CCC, B and BB. As observed, the simulations provide an increasing profile for the probabilities of defaults as time passes.

FIGURE 4: SIMULATION OF FUTURE DEFAULT PROBABILITIES



French agricultural data

The second example we give is based on more limited data. We investigate the default events of agriculture-related companies in France and link it to a very specific climate index. We have collected the historical series of companies having experienced a default over the past few years in France by sector of activities and have retained the one related to the agricultural sector ("Agriculture, forestry and fisheries"). Data come from Banque de France.[5] Secondly, we have set up an approximated yearly series of the total number of companies operating in France for the 2001-2024 period. Taking the ratio of these quantities has allowed us to derive an approximated historical series of probabilities of default for agricultural companies in France for the 2001-2024 period.

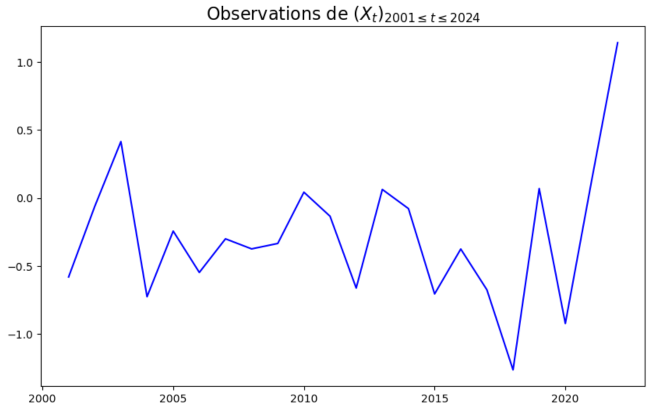
On those data, we proceed as above on the global data. We first extract the values of the state process X by minimising the likelihood in Equation (E_L).

3. ACPR. Scenarios and Main Assumptions of the 2023 Climate Stress Test Exercise. Retrieved 28 October 2024 from https://acpr.banque-france.fr/scenarios-et-hypotheses-principales-de-lexercice-de-stress-test-climatique-2023.

4. ACPR. Key Results of the Climate Exercise on the Insurance Sector. Retrieved 28 October 2024 from https://acpr.banque-france.fr/les-principaux-resultats-de-lexercice-climatique-sur-le-secteur-de-lassurance.

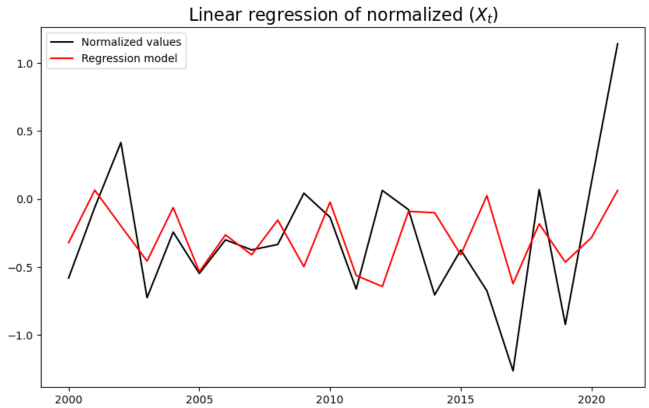
5. Banque de France (May 16, 2024). Business Failures Apr 2024. Retrieved 28 October 2024 from https://www.banque-france.fr/fr/publications-et-statistiques/statistiques/defaillances-dentreprises-avr-2024.

FIGURE 5: STATE PROCESS X



For deriving a model on the evolution of the state process $X = (X_t)_t$, we have chosen to work with the Soil Wetness Index (SWI), an index that takes values near (and even a little above) 1 when the soil is wet, and near (or a little below) 0 when the soil is dry. The values of the SWI are collected from the Météo France database:[6] they are given monthly and for 8,981 meshes covering the metropolitan French territory. To perform the linear regression representing the state process X, we work with yearly average values of the SWI over the whole 8,981 meshes. During the regression we thus have set $\widetilde{Y}_t = SWI_t$. The regression is displayed in Figure 6; the results are still quite satisfactory with $R^2 = 0.51$ and $\mathrm{MSE} = 0.21$. The p-values of significance tests for the coefficients $\alpha, \widetilde{\beta}$ and $\tilde{\nu}$ are, respectively, 0.005, 0.61 and 0.083.

FIGURE 6: LINEAR REGRESSION OF THE STATE PROCESS WITH RESPECT TO SWI



Again, now that the model is calibrated, simulations can be made. With the following experiment, we provide an example of the future evolution of probabilities of default on that sectorial example: we have set the default thresholds d_D being equal to its last observed value again and build two scenarios of evolution of the SWI: the first one is seeing the SWI decreasing at 2.5% per year; in the second one, it is increasing at 2.5% per year.[7] The simulated trends are consistent with intuition because, under the first scenario of an acceleration in the increase in drought, the probabilities of default do increase, quite significantly (Figure 7). Conversely, if the drought were to ease, the probabilities of defaults decrease on average in our simulations (Figure 8).

FIGURE 7: SIMULATION OF FUTURE DEFAULT PROBABILITIES UNDER DECREASE OF THE SWI

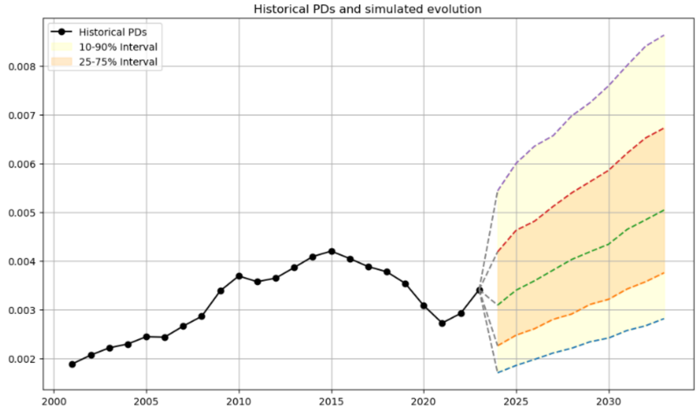
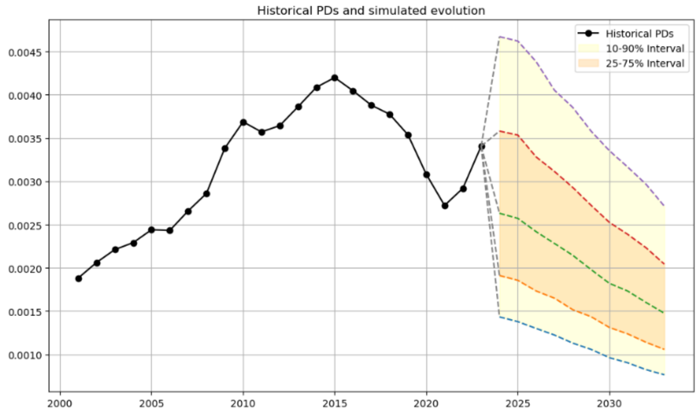


FIGURE 8: SIMULATION OF FUTURE DEFAULT PROBABILITIES UNDER INCREASE OF THE SWI



GOING FURTHER

This study paves the way to further work. First, this work could be reproduced with enriched datasets, including transition of ratings. Secondly, the fact the correlation coefficient ρ very often saturates to its lower bound during the calibration procedure could be further investigated.

6. Meteo France Public Data. Monthly Soil Moisture Index Data for the CatNat Device. Retrieved 28 October 2024 from https://donneespubliques.meteofrance.fr/?fond=produit&id_produit=301&id_rubrique=40.

7. Those two in-house scenarios are highly nonrealistic: in the worst scenario provided by Intergovernmental Panel on Climate Change (IPCC) Representative Concentration Pathway (RCP) 8.5, the SWI should decrease by 10% by 2050 over the full metropolitan territory (it is of course highly dependent on the geographical area).

A structural approach linking equities and credit spreads

OVERVIEW OF THE MODEL

One of the advantages of these models is that they enable us to consistently the model equity and debt data of a company. Starting from the original Merton model, Hull, Nelken and White (see MertonAndVol in References) have shown a way to compute the implied volatility of derivatives whose underlying basis is the equity index of the company, but such a model involves complex calculations and could lead to unstable results. An advanced framework, which allows us to link equities and credit spreads, is the so-called CreditGrades model, see CredGrad in References. CreditGrades is an extension of Merton's structural approach enhancing an initial limitation of the original model: short maturities are materially underestimated in Merton's model due to a constant default barrier. This barrier represents the level at which the company defaults: in CreditGrades, this barrier can be stochastic. Besides, some extensions of CreditGrades, especially the one proposed ExtCredGrad (see ExtCredGrad in References), use well-known dynamics to model the asset value of the company. For instance, one can model equity dynamics under Heston's model, where variance is modelled through a Cox, Ingersoll, Ross (CRI) process.

In this paper, we only consider a dynamic with stochastic variance and zero spot/variance correlation. Let V be the value of the firm's assets, S the value of the stock, r the time-dependent risk-free rate, W, W^{ν} two independent standard Brownian motions, κ the mean reversion speed, ν_∞ the mean reversion level of the variance process and ω the volatility of the variance process. By the very nature of these parameters, we impose $\omega > 0, \kappa > 0, \nu_\infty > 0$. The stochastic evolution of the process value and its variance is set to:

$$\frac{dV_t}{V_t} = r(t)dt + \sqrt{\nu_t}dW_t, (E_1)$$
$$d\nu_t = \kappa(\nu_\infty - \nu_t)dt + \omega\sqrt{\nu_t}dW_t^{\nu},$$

where it should be noted that $\mathbb{P}(\nu_t \geq 0) = 1$.

The stock dynamics can be deduced using the following relationship: $V_t = S_t + D(t)$, where D represents the level of debt of the company, deterministic and such that $D(t) = D(0)e^{\int_0^t r(s)ds}$. Hence, because $dD(t) = r(t)D(t)dt$, we deduce from (E_1) that:

$$\frac{dS_t}{S_t} = r(t)dt + \sqrt{\nu_t}\frac{S_t + D(t)}{S_t}\, dW_t.$$

Using this stochastic differential equation enables us to compute prices of derivatives whose underlying basis is the equity value of the company. Furthermore, we are interested in modelling the event of default of the given company. To do so, the model ExtCredGrad considers that the company will default when the stock value hits 0: this random date is denoted by $\tau := \inf\{0 \leq t \leq T, S_t \leq 0\}$. The conditional survival probability at date T knowing information at time $t \leq T$ is denoted by $Q(t,T) := \mathbb{P}(\tau > T \mid \mathcal{F}_t)$ (see Appendix for a discussion of this formula), with $(\mathcal{F}_t)_{t\geq 0}$ the filtration generated by the Brownian motion W. The credit spreads at time t for a maturity T is then usually approximated by

$$\varphi(t,T) := \frac{-\ln(Q(t,T))}{T-t}.$$

In our analysis, we are particularly interested in obtaining analytical formulas for two quantities: the credit spread and the call option price. It will allow us to calibrate the model either on spread data or option data.

To be able to calibrate the model, analytical formulas regarding both credit spreads and derivatives (call/put options) on the stock should ideally be derived. It turns out that the considered extension of the CreditGrades model offers several closed-form formulas.

The call price at time t for a maturity T at a strike price K and a spot level at S is given by:

$$C(t,T;S,K) = (D(T) + K)e^{-\int_t^T r(s)ds} Z_2(t,T,y,b)$$

The expressions of the functions Z_1 and Z_2 are given in the appendix.

Regarding the function Q that appears in the formula of the credit spread, it expresses as:

$$Q(t,T) = e^{y/2} Z_1(t,T,y).$$

Using these formulas, the calibration procedure is then quite standard. Let Ξ be the set of parameters of the model to be calibrated to market data:

$$\Xi := (S_0, D(0), \kappa, \nu_\infty, \nu_0, \omega),$$

and $(\varphi_i)_{i=1,..,N}$ the market spreads.

We need to solve the following optimisation problem:

$$\Xi^* = \operatorname{argmin}_\Xi \sum_{i=1}^{N} (\varphi(0,T_i) - \varphi_i)^2$$

The problem is of the same form for a calibration on implied volatilities. We compute the price of call options in our model, get the implied volatility by inverting the Black-Scholes formula and compare it to market value by measuring the sum of the square distance to each singular embedded option prices.

CALIBRATION ON MARKET DATA

We now propose to illustrate how the model can be used to include climate risk considerations.

We use market data related to credit spreads based on seven classes of ratings (AAA to CCC, as above, quoted on Euro Market as of 31 March 2024) gathering data from undertakings of all the sectors of activity and data relative to the implied volatility of call options (those comprising implied volatilities for relative strikes between 60% and 140%, for maturities between 1 and 10 years). We use the data to constitute benchmark calibrations (either by calibrating it on spreads or on implied volatilities; we don't need to try a joint calibration), to which we will compare our future experiments below.

Those market data will be used alongside the scenarios coming from the ACPR's second climate stress test.[8] We shock the term structure (uniformly) of corporate bonds with the values found in those scenarios and analyse how this deformation translates into deformation on the implied volatility surface.

The considered ACPR scenarios are the following:

- Long-term "Below 2°C": This scenario corresponds to an orderly transition towards a low-carbon economy, with policies implemented as of today.
- Long-term "Delayed transition": This refers to the scenario in which policies are not implemented immediately to counter the effect of climate change but will eventually be implemented in haste from 2030.
- Short-term: This is a short-term scenario (five-year horizon) that aims at capturing the macroeconomic effect of a sudden increase of physical risk (following a succession of extreme natural disasters in France).

CONVERTING CORPORATE BONDS SHOCKS INTO IMPLIED VOLATILITIES SHOCKS

We first calibrate again our model but now using stressed credit spreads (and unstressed market volatilities). The prescribed shocks under the "Delayed transition" scenario are given below:

HORIZON	SHOCK (BPS)
2025	+25
2030	+30
2035	+35
2040	+40

Those stresses are applied uniformly on the full credit curves. After having calibrated the parameters on the spreads, we can compute call option prices, and thus implied volatilities, with the obtained parameters; and we compare those implied volatilities to the ones obtained in the benchmark experiments, in which we only used market data (no shock). We provide below the differences observed on the full volatility surface when using the parameters for rating BB, for two horizons (2030 and 2040). Figures 7 and 8 display the difference between the volatility obtained from stressed spread curves and the volatility obtained from market spreads of the same rating

FIGURE 7: IMPACT ON VOLATILITY SURFACES FOR HORIZON 2030 – DELAYED TRANSITION

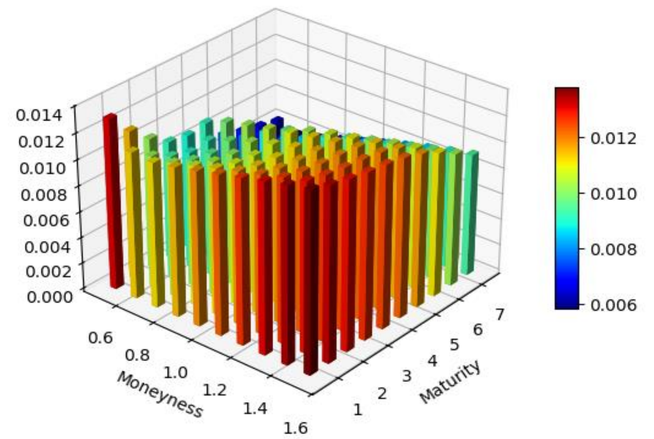
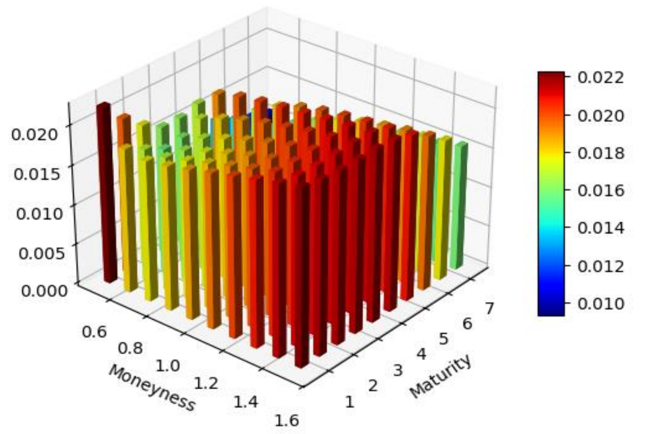


FIGURE 8: IMPACT ON VOLATILITY SURFACES FOR HORIZON 2040 – DELAYED TRANSITION



As expected, implied volatilities associated with stressed spreads are greater than volatilities associated with market spreads. Those impacts are distributed over the whole surface, mostly on small maturities and extreme moneyness. Besides, impacts have less magnitude as the horizon increases. Eventually, mean impact for 2030 is 10.25% whereas it is 17.74% for horizon 2040. It clearly shows a higher impact for further horizons.

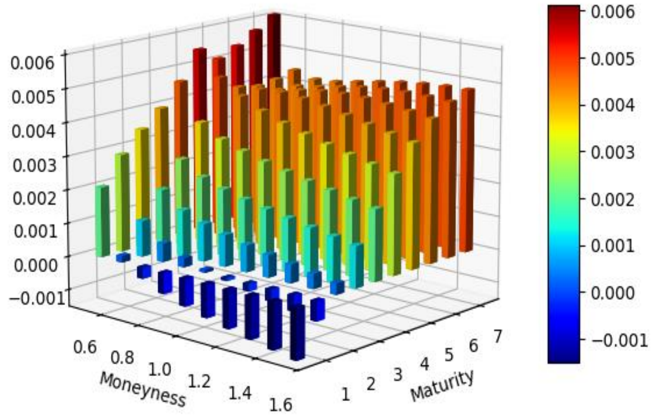
8. ACPR. Scenarios and Main Assumptions, op cit.

Secondly, we stress our spread curves using shocks for the "Below 2°C" scenario. Shocks used are:

HORIZON	SHOCK (BPS)
2030	-10
2040	-20

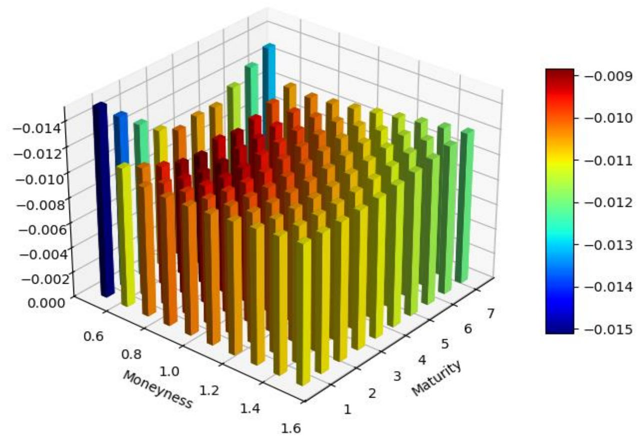
The impact for horizon 2030 is displayed in Figure 9.

FIGURE 9: IMPACT ON VOLATILITY SURFACES FOR HORIZON 2030 – BELOW 2°C



Here the change is less important than in the first scenario. Indeed, shocks on volatility surface are globally smaller than 1%; some impacts are negative. Again, this is something that was expected in this scenario. For "Below 2°C," the spread shocks are negative. The skew for short maturities seems to be reduced whereas skews for higher maturities have higher volatilities. For 2040, the impact on implied volatilities is presented in Figure 10.

FIGURE 10: IMPACT ON VOLATILITY SURFACES FOR HORIZON 2040 – BELOW 2°C



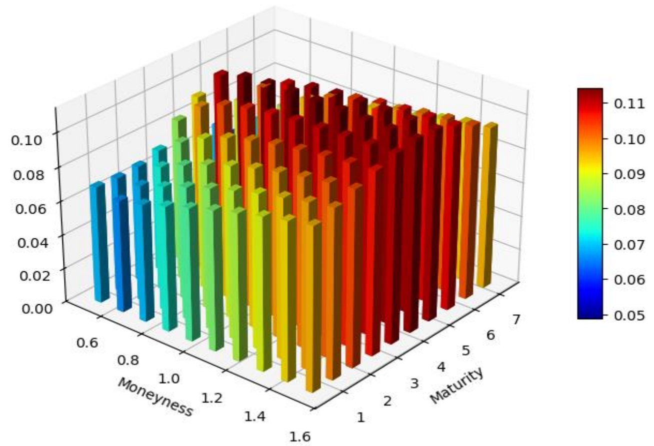
We observe now that stressed volatilities are uniformly smaller than market ones. In this scenario, mean impacts are 0.27% for 2030. It confirms our remark on Figure 9: shocks are quite small. For 2040, the mean impact is -1.05%. This is in line with expectations because credit spread shocks are negative.

We finally provide illustrations related to the last scenario, associated with short-term impacts. For this scenario, shocks depend on the maturity of the spreads. During the calibration, we have applied the shocks given by the ACPR and extrapolated them in a piecewise constant manner, so that all the maturities included in the calibration are stressed. In these values, as one can observe, shocks are significant:

HORIZON	MATURITY T	SHOCK (BPS)
2025	$T \leq 1$	+110
	$1 < T \leq 2$	+140
	$2 < T \leq 3$	+150
	$3 < T \leq 5$	+140
2027	$T \leq 1$	+110
	$1 < T \leq 2$	+150
	$2 < T \leq 3$	+150
	$3 < T \leq 5$	+140

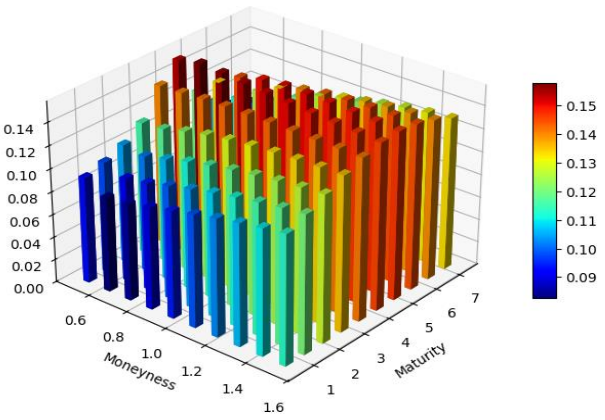
The resulting volatility gaps are shown in Figures 11 and 12.

FIGURE 11: SPREAD FOR HORIZON 2025 – SHORT-TERM SCENARIO



The model predicts a global and substantial rise of implied volatilities for the horizon 2025. This increase is particularly significant for large maturities (greater than three years). These results are confirmed by Figure 12.

FIGURE 12: SPREAD FOR HORIZON 2027 – SHORT-TERM SCENARIO



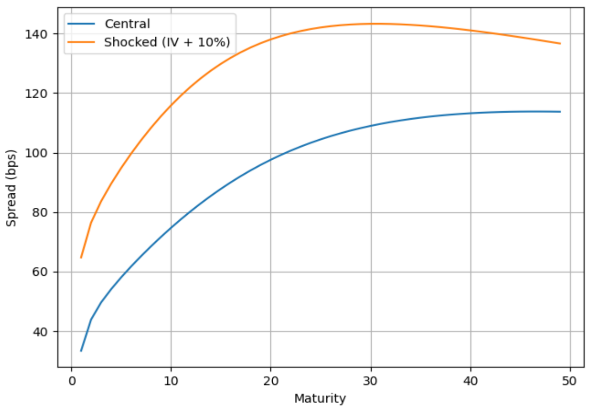
Here, the bar plot indicates an even higher rise of implied volatilities. As said before, this increase can be particularly observed on high maturities.

Finally, mean shocks for the last scenario are 9.22% for 2025 and 12.53% for 2026. Those values are close to the ones for the “delayed transition” scenario, but slightly lower.

CONVERTING IMPLIED VOLATILITIES SHOCKS INTO CORPORATE BONDS SHOCKS

So far, we have observed how a stress on spreads reflects on implied volatilities. To conclude this section, we provide an illustration of the other way around. The goal is now to observe how a shock on the whole volatility surface impacts the credit spreads. Here, the shock is applied relatively on the market-implied volatilities surface. We have chosen to add 10% on each volatility, that is, for all volatility σ in the surface, the shocked volatility $\tilde{\sigma}$ is given by: $\tilde{\sigma} = \sigma \times (1 + \delta)$ with $\delta = 10\%$. We then compared the produced spread curve to the original one implied from the market surface. Results are presented in Figure 13.

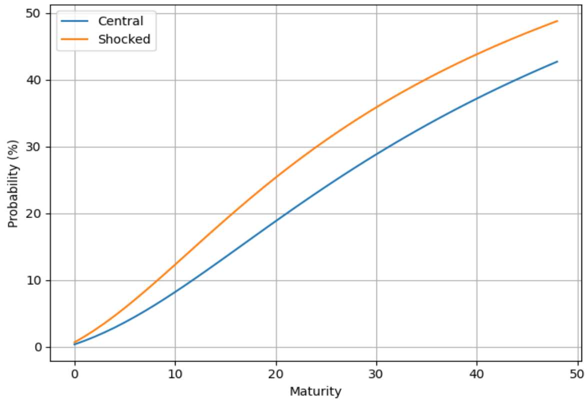
FIGURE 13: SPREAD CURVE FROM SHOCKED IMPLIED VOLATILITIES



Spreads are positively impacted by a rise of the volatility. Hence a rise of implied volatility leads to a rise of credit spreads, indicating that debt could be riskier and pay more interest.

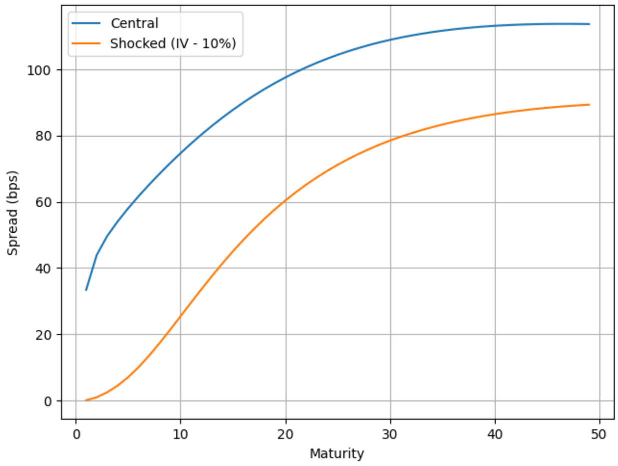
Another aspect to consider is the quantity representing directly default risk: the probability of default. Considering the previous hypothesis, default risk should be higher in this configuration. We now plot the initial default probability curve (default probability as a function of the maturity of the debt) against the shocked curve obtained by calibration of the model on the shocked surface.

FIGURE 14: DEFAULT PROBABILITY FROM SHOCKED IMPLIED VOLATILITIES



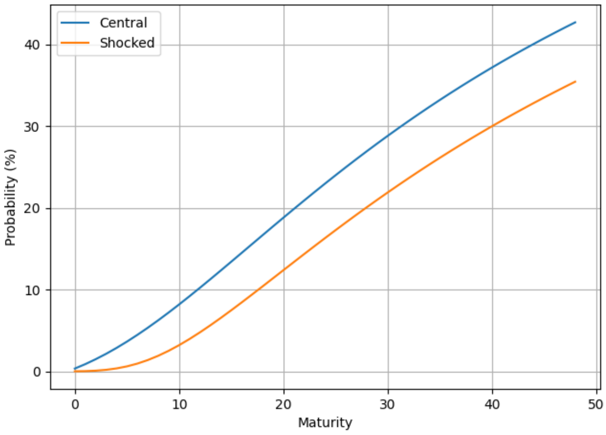
The same experiment was also performed on negative shocks. This time we take $\delta = -10\%$. Figure 15 presents credit spread from negatively impacted implied volatilities and Figure 16 shows the resulting probability of default.

FIGURE 15: SPREAD CURVE FROM SHOCKED IMPLIED VOLATILITIES



As expected, spread curve is negatively impacted as well. This indicates lower risks and thus lower default probabilities. This is confirmed by Figure 16.

FIGURE 16: DEFAULT PROBABILITY FROM SHOCKED IMPLIED VOLATILITIES



As expected, the new default probability curve is lower than the original one. We can conclude that implied volatility impacts directly default probability and credit spreads in a positive (for positive implied volatility shocks) or in a negative (for negative implied volatility shocks) way.

Conclusion

The final purpose of this paper is thus to help to design consistent climate stress tests. To do so, it describes in detail the simulation of scenarios (under both real-world and risk-neutral universes) that integrate transition climate risk in the paths of credit spreads and equities. The main novelty of the proposed methodologies is that those two risk factors are consistently impacted by the climate risk factor, which is useful because insurance (or other investors) may have in their portfolio equity and debt on a company. The described real-world approach is based on a systemic risk factor that embeds the climate change risk while the described risk-neutral method requires having a predefined (climate) stress test on either equities or credit spreads. Further work would consist of diffusing under risk-neutral measure a systemic risk factor that would affect both equities and spreads.

Appendix

SPREAD FORMULA IN CREDITGRADES

The price of a risky zero coupon (denoted by P^R) is given by the formula:

$$P^R(t,T) = \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^t r(s)ds}\,\mathbb{1}_{\{\tau>T\}} \mid \mathcal{F}_t\right],$$

where $\mathbb{Q}$ is the risk-neutral probability measure, τ is the previously defined default time, $\boldsymbol{\mathcal{F}}$ designates the filtration of the Brownian motion and $\boldsymbol{r}$ is the deterministic, time-dependent risk-free rate. We suppose the independence of the risk-free rate and the credit risk. Hence, we have:

$$P^R(t,T) = \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^t r(s)ds} \mid \mathcal{F}_t\right]\mathbb{E}^{\mathbb{Q}}\left[\mathbb{1}_{\{\tau>T\}} \mid \mathcal{F}_t\right].$$

The first term on the left is the price of a zero coupon at time t noted $P(t,T)$. The second term is the survival probability $Q(t,T)$. Because the credit spread is expressed as the difference between the yield of a risk-free zero coupon and the yield of a risky coupon, the expression for φ is:

$$\begin{aligned}\varphi(t,T) &= \frac{-\ln\left(P^R(t,T)\right) - \left(-\ln\left(P(t,T)\right)\right)}{T-t}\\ &= \frac{-\ln\left(P(t,T)Q(t,T)\right) + \ln\left(P(t,T)\right)}{T-t}\\ &= \frac{-\ln\left(Q(t,T)\right)}{T-t}.\end{aligned}$$

FUNCTIONS Z_1 AND Z_2

$$Z_1(t,T,y) = \frac{2}{\pi}\int_0^\infty \frac{e^{A(T-t,z)+B(T-t,z)\nu_t} z\sin(zy)}{z^2+1/4}dz,$$

$$y = \ln\left(\frac{S_t + D(t)}{D(t)}\right).$$

$$\begin{aligned}&Z_2(t,T,y,b)\\ &= e^y - e^b\\ &- \frac{e^{\frac{y}{2}}}{2\pi}\int_0^\infty \frac{\left[e^{A(T-t,z)+B(T-t,z)\nu_t}\cos(yz) - \cos\left((y-2b)z\right)\right]}{z^2+\frac{1}{4}}dz\,,\end{aligned}$$

$$\begin{aligned}y &= \ln\left(\frac{S_t + D(t)}{D(T) + K}\right) + \int_t^T r(s)ds\,,\ b\\ &= \ln\left(\frac{D(t)}{D(T) + K}\right) + \int_t^T r(s)ds.\end{aligned}$$

FUNCTIONS A AND B

Functions A and B are given by the following formulas:

$$A(\tau,k) = -\frac{\kappa\nu_\infty}{\omega^2}\left[\psi_+\tau + 2\ln\left(\frac{\psi_- + \psi_+ e^{-\zeta\tau}}{2\zeta}\right)\right],$$

$$B(\tau,k) = -\left(k^2 + \frac{1}{4}\right)\frac{1-e^{-\zeta\tau}}{\psi_- + \psi_+ e^{-\zeta\tau}},$$

$$\psi_\pm = \mp\kappa + \zeta,\ \zeta = \sqrt{\kappa^2 + \omega^2\left(k^2 + \frac{1}{4}\right)}.$$

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